The fifth volume of Advances in Mathematics Education focuses on an under-addressed area of research in mathematics education, namely early mathematical thinking and learning. Despite the groundbreaking work of Piaget that led to the formulation of developmental theories, interest in further developing neo-Piagetian models of learning has waned since the 1980’s. Three decades later, the community has come to realize that these developmental models do not take into consideration the sophisticated mathematical thinking that children are capable of, given the right mathematical activities to stimulate them into abstract reasoning.

The book, *Reconceptualizing Early Mathematics Learning*, edited by Lyn English and Joanne Mulligan presents studies that advance children’s mathematical learning in ways we did not think was possible. The chapters focus on notions of early algebra, statistical thinking, beginning numeracy as well as the advocacy for the kinds of learning that are important for the 21st century. Several of the chapters also address the professional development of teachers necessary to promote early mathematical learning experiences. The theoretical foundations of this work are set in Newton and Alexander’s chapter that surveys the state of the art. This is followed by empirical studies of Mulligan in Australia, Clements in the U.S. as well as alternative play-based classrooms of Wager. Data modeling is another theme explored by English with children in grades 1–3. Interdisciplinary approaches are also found in the work of Diefes-Dux that utilize model eliciting activities in art classrooms. The book provides a balance between theoretical foundations, empirical work with children that advance theories, as well as the importance of work with teachers to provide early mathematics learning and development.

An important feature to note in volume 5 is that the book series, *Advances in Mathematics Education*, has moved into topics not traditionally anchored in prior volumes of the connected journal, ZDM—The International Journal on Mathematics Education. This suggests that the series is open to research perspectives from the community that advance our field, without necessarily being anchored to ZDM.
We are deeply convinced that this book will make a strong contribution to the much needed diversity of theoretical advances in mathematics education.

Missoula, USA
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Perspectives on Reconceptualizing Early Mathematics Learning

Introduction

Lyn D. English and Joanne T. Mulligan

This edited volume emanated primarily from our concern that the mathematical capabilities of young children continue to receive inadequate attention in both the research and instructional arenas. Our research over many years has revealed that young children have sophisticated mathematical minds and a natural eagerness to engage in a range of mathematical activities. As the chapters in this book attest, current research is showing that young children are developing complex mathematical knowledge and abstract reasoning a good deal earlier than previously thought.

A range of studies in prior to school and early school settings indicate that young learners do possess cognitive capacities which, with appropriately designed and implemented learning experiences, can enable forms of reasoning not typically seen in the early years (e.g., Clements et al. 2011; English 2012; Papic et al. 2011; Perry and Dockett 2008). For example, young children can abstract and generalize mathematical ideas much earlier, and in more complex ways, than previously considered. Although there is a large and coherent body of research on individual content domains such as counting and arithmetic, there have been remarkably few studies that have attempted to describe characteristics of structural development in young students’ mathematics.

The title of this volume, *Reconceptualizing Early Mathematics Learning*, captures the essence of each chapter. Collectively, the chapters highlight the importance of providing more exciting, relevant, and challenging 21st century mathematics learning for our young students. The chapters provide a broad scope in their topics and approaches to advancing young children’s mathematical learning. They incorporate studies that highlight the importance of pattern and structure across the curriculum, studies that target particular content such as statistics, early algebra,

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and beginning number, and studies that consider how technology and other tools can facilitate early mathematical development. Reconceptualizing the professional learning of teachers in promoting young children’s mathematics, including a consideration of the role of play, is also addressed. Although these themes are diffused throughout the chapters, we restrict our introduction to the core focus of each of the chapters.

To set the scene, the opening chapter by Newton and Alexander provides an in-depth historical analysis of the changing and, at times paradoxical, nature of early mathematics learning. By conceptualizing how perspectives on early mathematics learning have taken shape over the past century through the impact of both internal and external forces, Newton and Alexander highlight the changing character of early mathematics learning over the last century. They explore psychological, socio-cultural, and neurophysiological developments that may have helped to shape these pedagogical trends in early mathematics education.

Emphasizing the importance of pattern and structure across the curriculum is the core feature of the chapters by Mulligan and her collaborators. Their classroom research with 4- to 8-year-old children reveals a focus on mathematical pattern and structure to be both critical and salient to young learners’ mathematical development. They demonstrate how their construct, *Awareness of Mathematical Pattern and Structure*, generalizes across early mathematical concepts, can be reliably measured, and is correlated with mathematical understanding. The construct can bring more coherence to our understanding of mathematical development and the design of effective pedagogical approaches. For example, we report on an evaluation study that demonstrates the positive impact of a Pattern and Structure Awareness Program (PASMAP) in the first year of schooling.

Another approach to advancing early mathematics learning is offered by Clements and Sarama with their learning trajectories tool, which forms the core of their conceptual framework for developing curricula and teaching strategies. The learning trajectories describe how children learn major topics in mathematics and how teachers can support that learning, while their framework details criteria and procedures for creating scientifically based curricula using learning trajectories.

An alternative perspective on supporting early mathematics learning is proposed by Wager with her focus on play-based classrooms. She explores the application of ‘focused’ instruction that minimizes teacher-centered practices and promotes play—teachers plan and build on children’s understanding, interests, and cultural practices, and recognize and respond to the mathematics that emerges in children’s play.

A focus on play also features in the chapter by van Oers, who addresses ways in which we might help children explore their actual play situations from the perspective of number. Specifically, the chapter describes how translating number related problems into thinking tools that are accessible for mathematical refinement (i.e. mathematizing) can occur meaningfully within the context of young children’s play. Such experiences are linked strongly to children’s learning to communicate about number in a coherent way, rather than by instructing them on number operations.

Providing opportunities for children to direct their own learning is also highlighted in English’s chapter, where she explores reconceptualizing young children’s
statistical experiences from the beginning years of formal schooling. Specifically, she addresses data modeling as a means of developing young children’s abilities to impose structure on complex data, detect relationships between seemingly diverse concepts and representations, and organize, structure, visualize, and represent data. Ways in which young learners engaged in these processes during a longitudinal study of data modeling across grades one to three are described.

Other ways of enriching learning opportunities are offered in the chapters that address advances in technology. Ginsburg et al. show how cognitive psychology can inform the design and evaluation of software for early mathematics learning, and how the resulting software can provide new approaches to evaluating learning and enhance basic cognitive research. With examples from their MathemAntics software program, the authors illustrate the affordances of computer technology in fostering transformative improvements in early mathematics education. The development of such software has the potential to elicit advanced mathematical thinking and reveal unexpected deviations from known developmental trajectories.

Likewise, Goodwin and Highfield demonstrate the rich learning experiences that interactive technologies can provide for the mathematics learning of 3–8-year-olds. Their exemplars demonstrate how the pedagogic design of technologies can have a substantial impact on young children’s development of basic mathematical concepts. In addition, Goodwin and Highfield provide evidence that different forms of multimedia offer unique opportunities for learners, whose responses challenge the widespread belief that young children are incapable of dealing with complex mathematical concepts.

Other didactic tools that have the potential to enhance early mathematical development include picture books, as seen in van den Heuvel-Panhuizen’s and Elia’s chapter. They present a framework of picture book characteristics that support kindergartners’ learning of mathematics, and examine three reading book techniques investigated in their research. A major conclusion of their research is reading picture books can support substantially children’s mathematical understanding and should thus have a significant place in the early curriculum. The use of picture books appears effective for a wide range of children including those of different ages, socio-economic backgrounds, and language and mathematical abilities.

An innovative, interdisciplinary approach to furthering early mathematical development is described in Diefes-Dux’ chapter. She considers how art education, which is typically viewed solely as an opportunity to explore creative thinking, can be a powerful partner in advancing young children’s problem solving, with a focus on mathematical modeling. The chapter describes a Draw-a-Monster activity that was created to adhere to design principles for a Model-Eliciting Activity and implemented in two art classrooms. Ways in which an activity of this nature can be linked to children’s learning in other domains including mathematics, language arts, and engineering are explored. Preparing young children for more complex modeling situations, appearing increasingly in their world, are discussed.

Enhancing teachers’ professional development is a core concern of the chapters by Warren and Miller, Papic, and Perry and Dockett. The first two chapters report on teacher programs designed to promote the early mathematics learning of disadvantaged, indigenous children. Warren and Miller’s program for teachers resulted in
improvements in their affective domain, with teachers becoming more confident in their ability to teach mathematics. These gains in teacher confidence led to improved pedagogical practices, and enriched mathematical content knowledge and instruction. In turn, these outcomes impacted positively on the children’s confidence and learning.

Papic reports on a series of studies aimed at improving young indigenous children’s learning opportunities, particularly in early algebra, patterning, and mathematical reasoning. The development of teachers’ pedagogical and mathematical content knowledge was also a core aim achieved through ongoing, supportive professional development. The program was geared towards the broader goal of closing the gap in numeracy achievement for Australian indigenous children in rural and regional early childhood settings. The studies outlined in their chapter provide empirical evidence that, through scaffolding teacher’s abilities to promote early mathematics learning, children’s development of sophisticated concepts and skills emerges. In particular, prior to formal schooling young children are capable of abstracting, generalizing, and explaining patterns and pattern structures.

Perry and Dockett’s chapter also addresses both teacher and student development, with a focus on findings from the Early Years Numeracy Project in South Australia. The development of a major artifact from the project, namely, the Reflective Continua, forms the focus of the chapter. Ways in which educators have used the Reflective Continua to stimulate the powerful mathematics learning of young children are reported. The Continua’s rich contribution to teacher development lies in its frameworks that guide educator reflections on children’s mathematical work and assist in the planning of future learning experiences.

In concluding, we thank the authors for their insightful and future-oriented perspectives on early mathematics learning and development. The book would not have been possible without their commitment to advancing the field; we hope their diverse collection of studies will provide a strong foundation for much needed future research.

References


Early Mathematics Learning in Perspective: Eras and Forces of Change

Kristie J. Newton and Patricia A. Alexander

One of the endlessly alluring aspects of mathematics is that its thorniest paradoxes have a way of blooming into beautiful theories. Phillip J. Davis (1964)

As the opening chapter in this important volume that looks deeply at the changing and somewhat paradoxical nature of early mathematics learning, our goal is to position those shifting perspectives within a historical framework. By conceptualizing how views of early mathematics learning have taken shape over the past century through the pushes and pulls of both endogenous (internal) and exogenous (external) forces, one can better grasp the re-conceptualization of mathematics learning conveyed within the ensuing chapters. There are perhaps few who would argue with the underlying premise of this book; that the character of early mathematics education has changed dramatically over the last century not only in terms of the pedagogical approaches to teaching young children, but also in relation to the content and goals of that instruction. However, the progression of that change may be less evident and, consequently, worthy of scrutiny.

Changes in complex domains such as early childhood mathematics rarely happen abruptly or without inducement. Rather, such transformations seemingly unfold over the course of many years in response to internal and external conditions. Here we endeavor to unearth those inducements, some of which arise more directly from within the community of researchers and practitioners invested in early mathematics teaching and learning. Other of those inducements can be situated within the broader educational and psychological communities, reflecting varied if not conflicting theoretical orientations toward human learning and development (see Fig. 1). Thus, in this chapter, we attempt to identify six particular periods or eras associated with

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mathematics learning in young children and seek to explore psychological, socio-cultural, and neurophysiological developments that may have helped to shape those eras.

Before we begin this historical overview of early mathematics learning, we want to state frankly that there is not exactness to the characterization we present. Historical analysis of this sort is characteristically inferential. Consequently, others who would engage in a comparative historical examination of the early mathematics literature or judge the endogenous and exogenous forces that were at work within each time period may reach different conclusions. Moreover, the boundaries between historical eras are neither rigid nor fixed. Here we looked at 20-year periods as meaningful, generational units for analysis, but the trends underway within the era do not simply begin or end at the preset time points. Further, our own interests and empirical foci, such as our investment in the study of learning theories and children’s understanding of fractions and their analogical reasoning, will undoubtedly color the perspectives we forge herein from the existing evidence.

Nonetheless, with these caveats in mind, the eras we identify derive from the theoretical and empirical literature of that period, and consider personages whose writings and thinking were particularly influential. We also signify some important events within mathematics, education, or the broader society that are anchored to these time periods. Also, we consider the views of children and the teaching of mathematics to children that were prevailing within each era, as well as the presence
of alternative or competing perspectives that may have signaled subsequent changes on the horizon.

For example, early childhood educators have traditionally advocated for learning through play, but views of the nature and purpose of play have varied markedly among educational researchers and practitioners. In part, these variations exist as a function of the changing beliefs about children’s cognitive capabilities and about the role of early educational experiences in enhancing the capabilities of young minds. They also mirror shifting psychological orientations toward learning and philosophical perspectives toward knowledge and knowing. An examination of the writings of such influential theorists as Dewey, Thorndike, Piaget, Vygotsky, Flavell, and Rogoff will serve to illuminate these shifting orientations toward young minds, mathematics, and the learning of mathematics. Drawing on these writings, we chart the course of early mathematics education in relation to these theoretical underpinnings, consider emerging trends, and address the implications for early mathematics research and practice.

Era of Experiential Learning (1900–1920)

Every historical analysis begins at some predetermined point in time. For our purposes, this analysis begins at the turn of the 20th century. So much of the world was undergoing change as the century dawned; civil conflicts in the United States and France had ended and the industrialization of much of Europe and the US was well underway. In terms of psychology, there was a growing interest in the nature of childhood itself—a concept that was not commonly considered prior to the industrialization of world powers—and an investment in developing mental faculties beginning at a young age (Elkind 1998). In the late 19th century, mathematics was considered important as a way to exercise the mental faculties in the developing mind (Sztajn 1995), and we see this notion carried forward into the early 20th century.

Those who were associated with early mathematics learning at the close of the 19th century, however, had not fully embraced the character of young children and their predisposition toward learning through free play that would become a feature of the next era. Rather, the available methods to teaching young children from that period were somewhat formal and structured in nature as perhaps best exemplified by the work of Friedrich Froebel. In particular, the techniques used by Froebel, who has been credited with introducing the concept of “kindergarten” into Western culture, involved concrete materials, such as geometric figures, that were deeply mathematical and which could be used to engage young children in the learning of mathematical concepts (Balfanz 1999). Froebel Gifts, as his educational materials were called, were very carefully devised and intended to be systematically used in the early childhood setting to foster particular ways of thinking and behaving. Froebel, as with Montessori who borrowed from his work, understood the role of “free work” or activities to teach the young. However, he saw these mathematical
activities not as an end in and of themselves but as important ways of exercising the developing mind of the child. Thus, Froebel’s approach could be described as rather formal and less spontaneous than we would see in the early 20th century in what we have labeled the Era of Experiential Learning.

By the early 1900s, the influence of Froebel was fading in favor of more holistic and less orchestrated conceptions of early childhood education (Balfanz 1999). Whereas Froebel’s kindergarten was rather structured in its treatment of mathematics, the more child-centered orientations of this era focused on the child as a social being. “Free work” was still central to early mathematics learning in this era, but the child was given increased freedom to choose and to freely explore the mathematically associated objects and activities that populated the learning environment. Mathematics was not directly the focus of learning in this setting, but was rather understood as manifestations of young children’s true interests that needed to be appropriately fed and actively nurtured through relevant and engaging experiences (Dewey 1903). Mathematical experiences existed but were informal in nature and embedded within children’s exploratory activities. In other words, the learning of mathematics was somewhat more incidental than intentional and the consequence of learning in experience rather than learning from experience (Saracho and Spodek 2009).

Influential Personages

No name is more associated with this Era of Experiential Learning than that of John Dewey, the pragmatist and the father of progressive education. In his formulation of progressive education, Dewey was influenced by Montessori’s idea of learning through activities and appreciated the efforts of Froebel to create learning environments for the young. Among the tenets of the progressive movement was Dewey’s often-expressed idea that education is the process of living and not simply a preparation of later life (Dewey 1900/1990). Toward that end, Dewey argued that the content of learning should derive from the children’s existing interests and draw meaningfully from the children’s life in the broader social community. Mathematics was not to be dealt with as isolated content nor used as mental calisthenics, but was to be experienced fully and naturally by children through hands-on, project-based activities that built on children’s existing interests and that pertained to activities (e.g., cooking, building) that were valued outside of school.

The focus on the teaching of mathematics to young children through engrossing experiences of value personally and socially coincided with the emergence of a new field, developmental psychology. Unlike the developmental psychology of today, this earliest manifestation of this field had more to do with systematic observation than experimental study. This focus can be clearly seen in the work of G. Stanley Hall (1907), considered the founder of developmental psychology, the first president of the American Educational Research Association, and the father of the child study movement. Perhaps best known for his fascination with peculiar and exceptional
children, Hall devised methods for the detailed documentation of children’s physical attributes and psychological behaviors.

The significance of developmental psychology, in general, and the child movement in particular was the now commonly accepted premise that young children are much more than miniature adults. Rather, they live and learn differently and those differences undergo systematic change over time. In terms of mathematics, this translated into critical questions about what the mind of the child was able to grasp mathematically and how best to harness the burgeoning knowledge about the physical and psychological development of children to teach them mathematical concepts and procedures appropriately (Alexander et al. 1989). Questions of developmental appropriateness and about how best to bring children and mathematics together reappear throughout the ensuing eras. For those of the Experiential Learning Era, that question was best answered by allowing young children to shape the educational agenda through the enactment of their interests and choices and by positioning the particulars of mathematical concepts and processes as background to the wants and desires of those children. Learning by doing remained the rule of the day, even as the uniqueness of young children was embraced.

Views of Children and the Teaching of Mathematics

In order to address growing concerns in the early part of the century about appropriate instruction in kindergarten, and in particular the appropriateness of Froebelian kindergarten, the International Kindergarten Union formed the Committee of Nineteen. A lack of consensus within the Committee meant that three reports on the content and goals of kindergarten were eventually issued, ranging from support of the Froebel method to endorsement of a more child-centered progressive orientation. Patty Smith Hill wrote the report supporting a child-centered approach, which ultimately won favor in academia (Beatty 1995). Following this report, play became a legitimate part of kindergarten programs and it was recommended that learning be guided by activities of interest to the child (Saracho and Spodek 2009).

Around this time, Margaret MacMillan established the first nursery school in England. Play was also an important part of this approach, but there was little concern for academic subjects. When academic subjects were introduced to older children, no particular approach to teaching was prescribed (Saracho and Spodek 2008). What was important instead to McMillan was the child’s health and hygiene, with a particular concern for poor and working class children. As a result, outdoor play and good air ventilation were incorporated into the program, to the extent that her early buildings were only partially enclosed and the children sometimes even napped outside (Beatty 1995). As nursery schools gained popularity over the next few decades, their focus would broaden to include the child’s general well-being and readiness for more formal learning in school.

Inspired by Dewey, Kilpatrick (1926) forwarded the Project Method, which engaged children in learning activities that were purposeful and practical. According
to Kilpatrick, children would be naturally stimulated to learn if they were provided with interesting experiences that involved them in the community. This followed the principles of progressive education wherein it was expected that children are naturally curious and interested in the world around them and that those curiosities and true interests included mathematical concepts and processes. There was also the presumption that the mind of the child was highly capable of dealing with mathematical concepts and procedures if those concepts and procedures were embedded in activities that the children valued and that they could reasonable pursue. Thus, when engaged in the Project Method, children may count or measure objects as they worked toward a larger goal, such as raising chickens (Saracho and Spodek 2008, 2009).

Mathematics was not emphasized in its own right, but it was expected that children would learn some mathematical ideas as they participated in the projects. We see similar orientations to mathematics learning in contemporary sociocultural theories of learning and development, such as Rogoff’s (1990) concept of legitimate peripheral participation. As with the aforementioned discussion, this overview of the Experiential Learning Era of early mathematics education introduces several themes about children and mathematics learning that will periodically reappear in our analysis. The first has to do with the perceived capacities of the young mind and whether the mind of the child is conceived as fertile ground for grasping basic mathematical concepts and procedures or not. The second has to do with the need to foreground the mathematical concepts or procedures or whether the mathematics should be embedded in socioculturally valued experiences or activities. For those in the Experiential Learning Era, children were perceived as highly self-directed and inquisitive learners who were able to acquire mathematical understanding if they were allowed to explore those ideas within the context of self-chosen, self-directed and socially valued activities.

**Competing Views**

Perhaps the most evident contrast to arise during this Experiential Learning Era was the all too familiar theme of traditional or basic skills education that has remained the counterpoint to progressive movements throughout the century. Specifically, Deweyan approaches to early childhood education were not the only ones that conflicted with the ideas forwarded by Froebel and his notion of kindergarten (Balfanz 1999). Another critic was Thorndike (1913), a behaviorist in terms of this theoretical orientation toward learning and development, whose work can be seen as the backbone for basic skills training and development. Thorndike purported that formal instruction in arithmetic was fruitless before second grade, and that even when mathematics was introduced, understanding was not a pre-requisite for acquiring mathematical skill (Baroody 2000). Rather than believing that mathematics could be learned incidentally through purposeful activities, Thorndike, who equated learning with behavioral change and manifestations, believed that mathematics must
be systematically structured and practiced and, thus, had no place in early childhood education. Thorndike, along with many other critics, thought that the early years should be focused instead on social development and health (Balfanz 1999).

**Era of Childhood Readiness (1920–1940)**

Two trends that appeared within the Era of Experiential Learning—learning through play and the focus on early childhood as a particular period of development—carried forward into the Era of Childhood Readiness. What distinguished this new era from the previous, as we will discuss, was the acceptance of mathematics as not solely as means to an end, but as a curricular end in and of itself. These trends combined together positioned the early educational years as a time to prepare the child for the more formal study of the domain of mathematics—to ensure that they were “ready” to think and perform mathematically in subsequent years.

As more students began to attend schools during the early twentieth century, an increased focus was placed on mathematics that was considered to be practical for the average person. This was especially true during the Great Depression, since limited availability of jobs kept many students in school for longer. The resulting increase in high school enrollment meant that more students were focused on training for jobs rather than college. Mathematics was de-emphasized, and in some cases, the mathematics requirements for graduation were removed altogether (Walmsley 2007). Instead, courses such as home economics, art, and physical education increased in popularity at the secondary level. Partly in response to the decreased focus on mathematics in schools, the National Council of Teachers of Mathematics (NCTM) was founded in 1920 (Austin 1921). Meanwhile, a parallel trend was occurring in early childhood education. In particular, mathematics in the early years was extremely limited. The focus shifted to play, imagination, physical movement, and social skills in part because this is what many educators felt young minds were able to cognitively and physically address (Saracho and Spodek 2009).

Several reasons contributed to this reality, including dominant theoretical perspectives during this era coupled with the changes occurring in the later grades. Mathematics was included as part of the first and second grades, but the amount of time spent on mathematics instruction was a fraction of the time spent on reading, language, and even recess (Balfanz 1999). Likely influenced by the continuing arguments by Thorndike and his adherents, it was determined that to focus any more specifically or directly on mathematics in these young years would not prove fruitful. Rather, drawing on the work in child study of the prior decades, it was held that educators needed to ascertain whether young children showed signs in their play and interactions with others that they were cognitively predisposed for formal instruction, including formal instruction in mathematics in the years to come.
Personages

Two contemporaries and colleagues warrant particular recognition for their role in shaping this Era of Readiness, Arnold Gesell and Frances Ilg. As has often been the case in the history, especially in these early years, Arnold Gesell came to education from a different field. He had trained to be a physician, but became enamored with questions of nature versus nurture and the role that each played in the development of children with disabilities. From decades of systematic research with Frances Ilg, he argued that nurture had a significant role to play in children’s developmental trajectory. Gesell and Ilg did not discount the power of nature but felt that there was much that could be done within the early years of life to stimulate cognitive capacity—to build on what nature had provided.

Gesell’s writing on *The Mental Growth of the Preschool Child* (1925), and *The Preschool Child from the Standpoint of Public Hygiene and Education* (1923), as well as his work with Ilg on the development of early childhood assessments served to justify the time as one of nurturing the young child—of readying them for the more formalized instruction in mathematics and other contents that would follow. His influence also extended beyond the educational community to parents concerned with child development and child rearing. This influence was largely due to his highly cited volume that documented early childhood milestones, *An Atlas of Infant Behavior* (1934) and to the two guides for child rearing that he coauthored with Ilg, *Infant and Child in the Culture of Today* (1943), and *The Child from Five to Ten* (1946).

By the close of this era, many regarded Gesell as the foremost authority on child rearing and child development. Not only did he argue strongly for the influence of early nurturance at home, but he also became an advocate for a nationwide nursery school system that could provide the early stimulation and support that he promoted in his writings to educators and to parents. And it was these strongly held beliefs in the importance of readiness to later development that mark this era, particularly when coupled with the Thorndikian perspective that training in mathematics should be reserved for later elementary and not attempted within the early grades.

Views of Children and the Teaching of Mathematics

Views of young children and the teaching of mathematics during this era were shaped by an emerging interest into the inner workings of the human mind (cognition) in relation to the behavioral indicators of capability (behaviorism). In line with Thorndike’s work, some theorized that formal instruction in mathematics was unnecessary—and perhaps even harmful—in the early years and should be delayed until the child was in a formal school setting (Balfanz 1999). As a result, the mathematics curriculum was limited in the early years. Nursery schools, which were popularized during this time, held little regard for academic subjects in general and even